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Numerical processing profiles in children with varying degrees of arithmetical achievement

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ABSTRACT

Recent studies show basic cognitive abilities such as the rapid and precise apprehension of small numerosities in object sets ("subitizing"), verbal counting and numerical magnitude comparison significantly influence the acquisition of arithmetic and continues to modulate more advanced stages of mathematical cognition. Additionally, children with low arithmetic achievement (LAA) and Developmental Dyscalculia (DD) exhibit significant deficits in these cognitive processes. Nevertheless, the different cognitive profiles of children with varying degrees of numerical and arithmetic processing deficits have not been sufficiently characterized, despite its potential relevance to the stimulation of numerical cognition and the design of appropriate intervention strategies. Here, the cognitive profiles of groups of typically developing children, children with low arithmetical achievement and DD, exhibiting typical and atypical subitizing ability were contrasted. The results suggest that relatively independent neurocognitive mechanisms may produce distinct profiles at the behavioral level and suggest children with low arithmetic performance exhibiting atypical subitizing abilities are not only significantly slower, but rely on compensatory mechanisms and strategies compared to typical subitizers. The role of subitizing as a correlate of arithmetic fluency is revised in the light of the present findings.

1. Introduction

It has been suggested that basic numerical abilities such as the estimation of the number of items in small sets of objects "at a glance" ("subitizing") and the ordering of sets by their "numerosity" (cardinality) are predictors of arithmetic performance at school (Halberda, Mazzocco, and Feigenson, 2008; Reigosa-Crespo et al., 2013). These capacities seem to underlie the understanding of the relationship between sets and their numerosities; as well as the types of manipulations that may or may not affect the numerosity of sets.

Recent cross-sectional and longitudinal studies on numerical cognition have provided evidence that these basic numerical capacities influence the subsequent acquisition of arithmetic and continue to modulate more advanced stages of mathematical cognition (Koontz and Berch, 1996; LeFevre et al., 2010; Schleifer and Landerl, 2010; Kroesbergen, van Luit, van de Rijt, van Lieshout, and van Loosbroek, 2009). Likewise, selective deficits of these basic numerical abilities have been reported in adults with brain lesions that produce Acalculia (Dehaene and Cohen, 1995; Seron et al., 1991) and in children with Developmental Dyscalculia (DD), in presence of clinical-genetic risk factors that can be associated with the disorder (Bruandet, Molko, Cohen, and Dehaene, 2004; Simon et al., 2005) and also in typically developing children with DD without clinical-genetic risks accounting for the presence of this neurodevelopmental disorder (Koontz and Berch, 1996; LeFevre et al., 2010; Reeve, Reynolds, Humberstone, and Butterworth, 2012; Reigosa-Crespo et al., 2013; Rousselle and Noël, 2007; Träff, Olsson, Östergren, and Skagerlund, 2016).

According to the Diagnostic and Statistical Manual of Mental Disorders of the American Psychiatric Association (DSM-5, American Psychiatric Association, 2014) DD is characterized by significant difficulties mastering number sense, number facts, or calculation (e.g., poor understanding of numbers, their magnitude and relationships; counting on fingers to add single-digit numbers instead of recalling the math fact; lose track of arithmetic computation and switch procedures) or difficulties with mathematical reasoning (e.g., severe difficulties applying mathematical concepts, facts, or procedures to solve quantitative

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problems). Children's performance must be quantifiably below what is expected for the individual's chronological age, must not be caused by poor educational or daily activities or by intellectual impairments and to cause significant interference with academic or occupational performance, or with daily life activities, as confirmed by individually administered standardized achievement measures, and comprehensive clinical assessment. The learning difficulties described usually begin during school-age years but may not become apparent, until the assessment or academic demands exceed the individual's limited capacities, as in the case of timed tests, reading or writing lengthy complex reports with a tight deadline or excessively heavy academic loads.

The prevalence of specific learning disability in mathematics ranges from 2.5 to 6.4% in the general school population (Gross-Tsur, Manor, and Shalev, 1996; Lewis, Hitch, and Walker, 1994; Reigosa-Crespo et al., 2012; Shalev and Gross-Tsur, 2001). There have been significant gains over the past decade, regarding the study of the origin of DD and the brain foundations of the disorder. Nevertheless, outstanding questions about the cognitive characterization of DD remain unanswered. This fundamental aspect would substantially contribute to the design of appropriate intervention strategies for children exhibiting the disorder and for the numerical processing stimulation in typically developing children.

It has been recently suggested that it is important to differentiate between “primary” and “secondary” DD (Kaufmann et al., 2013; Price and Ansari, 2013). According to this approach, primary DD is characterized by severe deficits in numerical or arithmetic processing associated to underlying biological factors. On the other hand, secondary DD, is characterized by impaired numerical processing that can be explained by deficits in non-numerical processes, such as attention or working-memory capacity (Kaufmann et al., 2013; Wilson and Dehaene, 2007), verbal reasoning (Geary, Hamsom, and Hoard, 2000) and visual-spatial abilities (Geary, Hoard, and Hamson, 1999; Robinson, Menchetti, and Torgesen, 2002). Furthermore, it has been pointed out that even primary DD may be subject to further subtyping, considering the basic numerical capacities involved in numerical cognition and its interactions with domain-general cognitive processes contributing to mathematical achievement.

In fact, several hypotheses have been put forward to explain domain-specific numerical and arithmetical processing impairments in DD. Regarding exact enumeration of small sets of objects, it has been reported that children with DD have a restricted subitizing range (up to 3 objects) (Andersson and Östergren, 2012; Desoete and Grégoire, 2006; Furman and Rubinsten, 2012; Moeller, Neuburger, Kaufmann, Landerl, and Nuerk, 2009), compared to typically developing children (4 objects) (Ashkenazi, Mark-Zigdon, and Henik, 2013). Also, DD children' subitizing slopes have been reported to be significantly steeper compared to those of typically developing controls, indicating a dysfunctional subitizing mechanism (Schleifer and Landerl, 2011). Initially, small objects enumeration was proposed to be explained by the single estimation process hypothesis: the same process was involved in small and large numerosities' estimation (Dehaene and Changeux, 1993; Feigenson, Dehaene, and Spelke, 2004; Gallistel and Gelman, 1991). However, this hypothesis was challenged by the fact that there were described dissociations between small and large numerosity estimation processes both in adults and children. More recently, subitizing has been accounted for by a visuospatial object-based attention system, able to keep track of up to 4 objects; called the object-tracking system (OTS) (Desoete and Grégoire, 2006; Piazza, 2010; Piazza et al., 2010; Wilson and Dehaene, 2007). Additionally, some studies have shown subitizing estimation measures correlate with visual working memory capacity (Formoso, Barreyro, Jacubovich, and Injoque-Ricle, 2017), however, others have reported subitizing accounts for individual variability in arithmetic achievement after controlling for other domain-general cognitive processes (Reigosa-Crespo et al., 2012) and altogether, the evidence suggest an object-based attention system is responsible for subitizing and that it is dissociated from other dedicated domain-specific numerosity representation systems (Zorzi, Stoianov, and Umilta, 2005).

Concerning larger numerosities estimation, a predominant hypothesis is that numerical or arithmetic deficits in DD are originated from a deficit in the Approximate Number System (ANS), an innate core system specialized in the representation of quantities in an approximate manner; which has also been suggested as the foundation onto which the symbolic number system is mapped (Dehaene, 1997; Gelman and Butterworth, 2005; Piazza, 2010).

On the other hand, the numerosity-coding hypothesis (Butterworth, 2010; Zorzi et al., 2005) states that DD is caused by deficits in a core system dedicated to the exact representation of the discrete elements of sets. This number system allows the representation of large numerosities (above 4 objects) in an exact manner, much like the object tracking system for small sets of objects. Data showing non-symbolic and exact enumeration difficulties in DD children has been recently reported in a study of Chinese children (Chan, Au, and Tang, 2013).

Finally, according to the access deficit hypothesis (Rousselle and Noël, 2007), DD is caused by a defective connection between numeric symbols (Arabic digits) and the analogic numerical magnitude representations. Thus, DD would not result from a deficit in the innate ANS (Rousselle and Noël, 2007; Wilson and Dehaene, 2007). Evidence has been reported by De Smedt and Gilmore (2011), Landerl and Kölle (2009), Rousselle and Noël (2007), Castro and Reigosa (2011), and Castro, Reigosa, and González (2012), showing children with DD exhibited significant impairments in symbolic number comparison, but typical non-symbolic number comparison.

These reports suggest primary DD is, in fact, a heterogeneous disability, comprising different neurocognitive subtypes. In a more general view, it has also been described that arithmetical disabilities may be associated with any one of, or combination of, a number of heterogeneous deficits (Baddatz, Kuhn, Holling, Moll, and Dobel, 2017). Following this view, the present study aims to elucidate whether deficits in specific basic numerical abilities, would produce different and identifiable behavioral profiles. The present study aims to address this question by examining the numerical processing profiles of groups of children within an ample range of numerical and arithmetical performance, including typically developing children, children with low arithmetical achievement (LAA group) and children with DD. Additionally, the particular contribution of subitizing to arithmetic achievement will be examined, by identifying and comparing groups of children exhibiting typical and atypical subitizing ability among the different groups classified using arithmerical achievement and numerical processing standardized tests.

In order to produce both, a quantitative and qualitative characterization of the spared and impaired processes in children with DD, computerized exact enumeration and symbolic comparison item-timed tasks were used in order to assess small numerosities representation, counting and symbolic numbers comparison. These tasks were meant to exceed the potentially damaged individual capacities in the case of children with arithmetical or numerical processing deficits (Butterworth, 2003) and appropriately describe their cognitive profiles.

We predict children with LAA or DD with impaired subitizing abilities would exhibit different reaction time patterns as a function of numerosity within the subitizing range, compared to controls; despite their different degrees of numeric processing impairment. These differences may include significantly slower RTs and reduced subitizing range. Additionally, no differences are predicted in counting and number comparison processes across groups.

2. Methods

The sample included 200 children from 7 to 15.58 years old, at risk of low arithmetic achievement in a nonstandardized curriculum-based screening of mathematics attainment (see details below) that was group-administered at the beginning of the school year; who underwent

neurocognitive assessment in basic numerical abilities and mental arithmetic tasks (using the Basic Numeric Battery; Reigosa-Crespo et al., 2012) and which data was still available for further analysis.

3. Participants and procedure

Each child was assessed individually in a single testing session of approximately 20 min. The assessment was conducted in quiet and illuminated rooms at the schools. Written consent from all parents was obtained, and all participants provided written assent for assessments.

4. Materials

4.1. Mathematical attainment test (MAT)

The MAT is a nonstandardized curriculum-based measurement. It is the official mathematics achievement test designed by the Central Institute of Pedagogical Science (ICCP) from the Cuban Ministry of Education to assess children academic attainment regarding grade appropriate benchmarks. It comprised eight computational problems by grade (second to ninth), representing a proportional sampling of the computation skills within the national curriculum. Total score was up to 8 (one score was awarded for each problem performed correctly).

4.2. Basic numerical battery (BNB)

BNB (Reigosa-Crespo et al., 2012) is a battery of item-timed computerized tests, including two numerical capacity tests: dot enumeration and numerical magnitude comparison and a test of mental arithmetic fluency. Each test included practice trials in order to ensure the subjects understood the instructions. The children always responded by pressing the corresponding key (thus, misses were not measurable). Only the keys of the numeric pad were available for response (except for the simple reaction time task).

4.2.1. Simple reaction time

This task was included as a baseline measure of processing speed. This measure was not analyzed by itself; the reaction times (RT) on the following three computer tests described below were adjusted by subtracting simple RT recorded in the simple reaction time task from RT on each test. Children were asked to press the space bar as soon as they saw a square in the center of the display. Variable inter-stimulus presentation time was employed (500–1.500 ms). Twenty trials were presented. Five practice trials were given before the beginning of the test. RTs were recorded with millisecond precision.

4.2.2. Numerical capacity tests

4.2.2.1. Dot enumeration. Randomly arranged dots (numerosities ranging 1–9) presented on the computer display. Children were asked to enumerate the quantities and to respond as quickly as possible without making mistakes. RTs and errors were recorded by pressing the key corresponding to the number of dots enumerated. Eighteen trials were presented altogether, with each number from 1 to 9 being presented twice in a pseudorandom order, with the proviso that no item occurred twice in succession. Five practice trials were included before the beginning of the test.

4.2.2.2. Numerical magnitude comparison. Children were presented with two digits (numerosities 1–9) on the computer display, one to the left and one to the right of the screen, and they were asked to compare the magnitude of numbers from left to right (e.g., $5 < 7$, $7 > 5$). The numerical distance between the pairs was manipulated (numerical distance ranging 1–8). The response keys were "1" for " $>$," "2" for " $>$ " and "3" for "=". Thirty-six trials were presented in pseudorandom order. Five practice trials were included before the beginning of the test. Reaction times and errors were recorded.

4.2.2.3. Mental arithmetic. Fifteen simple additions, 15 subtractions, and 15 multiplications were presented in three separate blocks. All including single-digit numbers (Andersson and Östergren, 2012; Ansari, Lyons, van Eimeren, and Xu, 2007; Ashkenazi, 2016; Ashkenazi et al., 2013; Baddatz et al., 2017; Bruandet et al., 2004; Burr, Turi, and Anobile, 2010; Butterworth, 2003). No ties (e.g., $3 + 3$, 5×5) were included, and items were not repeated. Items were presented on the computer screen in the form "2 + 4." Two practices trials per operation were included before the beginning of the test. Children were asked to type in the answer as soon as possible without making mistakes. RT was measured with the first key stroke. Errors were also recorded.

Individual Z scores were calculated per task, following the procedures described by Reigosa-Crespo and collaborators (2012).

The BNB's presentation paper reported low sensitivity (0.27) but very high specificity (0.98), using Efficiency on Mental Arithmetic as the gold standard. Consequently, a positive result in BNB was itself fair at confirming DD (predictive positive value was 0.74), and a negative result was very good at reassuring that a child does not have DD (negative predicted value was 0.86) (Reigosa-Crespo et al., 2012).

4.3. Intellectual capacity

Scores from the Blocks Design and Vocabulary subtests of the reduced version of the Wechsler Intelligence Scale for Children (WISC-R; Sattler, 1982) were used as measures of non-verbal and verbal IQ, respectively. The WISC includes two subtests: the Vocabulary and Blocks Design subtests. The IQ calculated using these two subtests have been reported to correlate 0.98 with the global IQ produced by the whole scale.

4.3.1. Classification procedure

The children were included in three different groups, based on their results in the BNB tasks and the results in the WISC-R; (Sattler, 1982): 1) Low Arithmetic Achievement Group (LAA): children exhibiting significant difficulties (Z scores > 1.98) in the Basic Calculation Task from the BNB and IQ greater than or equal to 80 when evaluated with the WISC-R; 2) DD group: children exhibiting significant difficulties (Z scores > 1.98) in the Dots Enumeration or the Digits Comparison tasks from the BNB and IQ greater than or equal to 80 when evaluated with a reduced version of the WISC-R together with spared arithmetical performance ($Z \leq 1.98$ in the Basic Calculation Task) and, 3) the control group: (Z scores ≤ 1.98) in the Dots Enumeration, the Digits Comparison and the Basic Calculation Task from the BNB and IQ greater than or equal to 80 in the WISC-R.

In order to identify children with typical and atypical subitizing performance across the previously diagnosed groups; an independent methodology was employed. Individual reaction times corresponding to the detection of dots in the subitizing range (1 to 3 dots) were corrected by processing speed and employed to calculate the difference in RT in the detection of 3 vs. 1 dot. The subitizing and counting patterns of children exhibiting negative differences for 3–1 dots were visually inspected and finally removed from the following analysis, as they seemed too noisy to reflect consistent performance in the task. The new difference measure in the resulting sample was analyzed by using the Expectation-Maximization (EM) method (Vlassis and Likas, 2002) implemented in Matlab7. With an a priori definition of how many subpopulations may exist in a general distribution, the EM method determines the value of the mean and the SD of each of the identified subdistributions. In this case, two subpopulations were defined a priori: typical and atypical subitizing patterns.

To characterize the resulting subsamples, an analysis of covariance (ANCOVA) was conducted on the RTs for the detection of 1 to 3 dots (subitizing range), 6 to 9 dots (counting range), and for the RT for the digits comparison task, ranging from close to far numerical distances (), including age as covariate and EM classification as between factor.

Finally, using the new classification, children exhibiting

significantly high Z scores in Dots enumeration were removed from the analysis and the remaining children were re-distributed in six groups:

1) Controls with typical subitizing performance (Controls_T): controls according to the BNB classification and typical according to the EM classification.
2) Controls with atypical subitizing performance (Controls_A): controls according to the BNB classification and atypical according to the EM classification.
3) Low Arithmetic Achievement Group with typical subitizing performance (LAA_T): children with low arithmetic achievement according to the BNB classification and typical according to the EM classification.
4) Low Arithmetic Achievement Group with atypical subitizing performance (LAA_A): children with low arithmetic achievement according to the BNB classification and atypical according to the EM classification.
5) DD Group with typical subitizing performance (DD_T): children with low digits comparison abilities and spared arithmetic achievement according to the BNB, together with a classification of typical subitizers, according to the EM classification.
6) DD Group with atypical subitizing performance (DD_A): children with low digits comparison abilities and spared arithmetic achievement according to the BNB, together with a classification of atypical subitizers, according to the EM classification.
7) Depending on sample size, the new groups' enumeration and magnitude comparison RTs were included in several ANCOVAs to identify potentially different behavioral profiles between groups. The new classification was included as between group effect, the numerosities (subitizing range: 1 to 3; counting range: 6 to 9) and numerical distances () were included as within groups effects and age was included as a covariate in all analysis.

5. Results

5.1. Classification according to the BNB diagnostic criteria

One hundred and thirty nine children were classified as controls, 28 children were included in the LAA group and 23 children in the DD group (8 DD children exhibited significantly low digits comparison (DD_DC) and the remaining 15 showed significantly low dots enumeration compared to the normative sample (DD_DE)). Additionally, several comorbidities were detected in the rest of the sample. See Table 1 for sample details. One DD child exhibited both significantly low dots enumeration and digits comparison in the presence of spared basic calculation abilities. The remaining nine children showed comorbidities between low arithmetic achievement and DD. See Table 1 for sample details.

ANCOVAs conducted on intellectual capacity, including group as between factor and age as continuous predictor, showed significant effects group $F(3, 185) = 9.6$, $p < .001$, Partial eta-squared = 0.13 and age, $F(1, 185) = 10.8$, $p = .0012$, Partial eta-squared = 0.06. Controls exhibited significantly higher intellectual capacity compared to the LAA group. There were found no significant differences compared to the DD-DC or the DD-DE groups (see Table 2 for details). However, note all children included in the sample exhibited IQ ≥ 80. There were no statistically significant differences among the LAA, the DD_DC and the DD_DE groups.

On the other hand, ANCOVAs conducted on arithmetical attainment including group as between factor and age as continuous predictor, showed significant effects of group $F(3, 185) = 254$, $p < .001$, Partial eta-squared = 0.80 and age, $F(1, 185) = 11$, $p = .0011$, Partial eta-squared = 0.06. Controls exhibited significantly higher raw scores compared to the rest of the groups. No significant differences were found between the rest of the groups.

Table 1
Sample description: sample size by grade, according to the BNB classification.

Groups	N (F)	Grade (Age range)							
		2nd	3rd	4th	5th	6th	7th	8th	9th
Sample	200 (106)	12 (7)	15 (8)	22 (13)	19 (11)	18 (9)	13 (5)	51 (31)	50 (22)
Controls	139 (74)		5 (2)	20 (11)	16 (10)	14 (6)	10 (4)	30 (21)	44 (20)
LAA	28 (15)	8 (4)	7 (4)	1(0)	1 (1)	3 (3)		6 (2)	2 (0)
DD_DC	8 (6)	2 (2)			1 (0)			4 (3)	1 (1)
DD_DE	15 (8)		1 (1)		1 (0)		1 (1)	9 (5)	3 (1)
DD_DE_DC	1 (0)						1 (0)		
LAA_DD_DC	6 (2)	2 (1)	1 (0)	1 (0)		1 (0)		1 (0)	
LAA_DD_DE	2 (1)		1 (1)					1 (1)	
LAA_DD_DE_DC	1 (0)						1 (0)		

Note: LAA: Low Arithmetic Achievement Group, DC: Developmental Dyscalculics exhibiting impaired Digits Comparison, DD_DE: Developmental Dyscalculics exhibiting impaired Dots Enumeration, DD_DE_DC: Developmental Dyscalculics exhibiting impaired both, Dots Enumeration and Digits Comparison, LAA_DD_DC: Developmental Dyscalculics exhibiting impaired Digits Comparison and Low Arithmetical Achievement (mental calculation task), LAA_DD_DE: Developmental Dyscalculics exhibiting impaired Dots Enumeration and Low Arithmetical Achievement, LAA_DD_DE_DC: Developmental Dyscalculics exhibiting impaired both, Dots Enumeration and Digits Comparison and also Low Arithmetical Achievement.

Table 2
Sample description: cognitive assessment by group according to the BNB classification.

Groups	Sample Size N (F)	CI Mean (SD)	MAT Mean (SD)	DE Mean (SD)	DC Mean (SD)	BC Mean (SD)
Controls	139 (74)	104 (13.8)	7.0 (1.2)	2259 (621)	1621 (605)	2611 (1013)
LAA	28 (15)	93.0 (9.4)	1.9 (1.0)	3596 (968)	2853 (1018)	15,699 (14352)
DD_DC	8 (6)	97.9 (15.0)	2.4 (1.2)	3141 (1125)	6160 (3255)	6118 (3364)
DD_DE	15 (8)	96.5 (10.5)	2.1 (0.9)	4284 (905)	2196 (1146)	4106 (929)

Note: LAA: Low Arithmetic Achievement Group, DC: Developmental Dyscalculics with impaired Digits Comparison, DD_DE: Developmental Dyscalculics with impaired Dots Enumeration.

5.2. Expectation maximization analysis

Seventeen children exhibiting noisy reaction time patterns in the exact enumeration task were excluded from all following analysis. In the general distribution of the difference between the reaction times for 3 compared to 1 dot, were $M = 414$ ms and $SD = 263$ ms for subpopulation 1 ($N = 125$); and $M = 1253$ ms and $SD = 734$ ms for subpopulation 2 ($N = 58$). The means of the subpopulations were statistically significant when performing a *t*-test for independent samples assuming a normal distribution ($p < .001$).

As expected, an ANCOVA conducted in the subitizing range's reaction times including group as between factor, numerosity as within factor and age as continuous predictor showed the expected group effect, showing the atypical subitizing (henceforth, non-subitizers), significantly higher reaction time, $F(1, 180) = 33.5$, $p < .001$, Partial eta-squared = 0.15. Significant numerosity effect was also found, F(2,

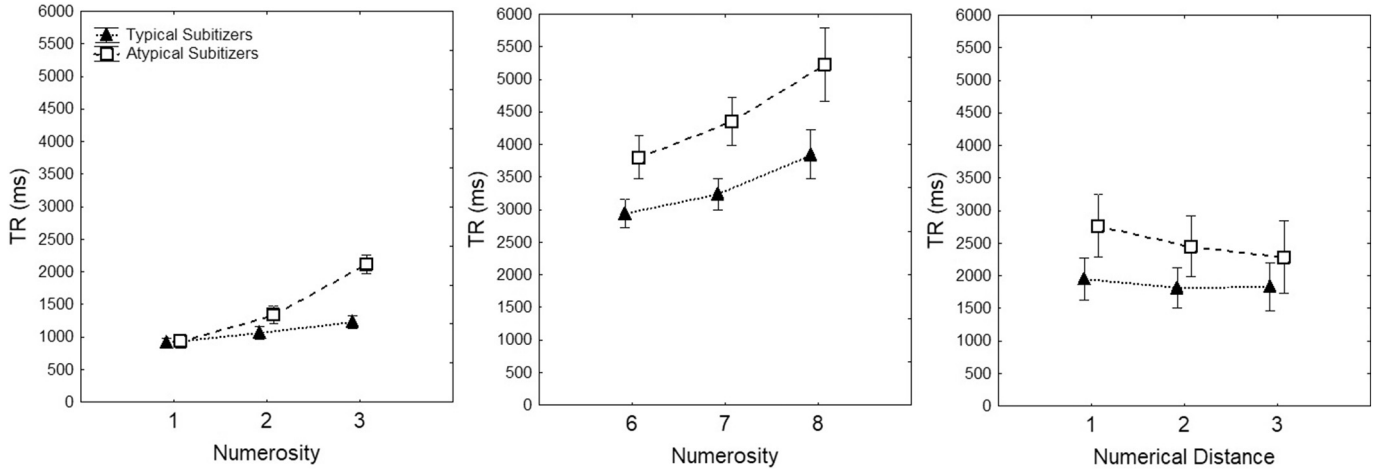


Fig. 1. Subitizing, counting (dot enumeration) and numerical magnitude comparison (digit comparison) processes in the typical (N = 125) and atypical subitizing group (non-subitizers, N = 58), according to the Expectation Maximization classification. The Atypical subitizers showed significantly higher corrected reaction times in all tasks. Significant numerosity effects were found in the subitizing and counting ranges. However, no significant distance effect was confirmed. Note the significant interaction between group and numerosity in the subitizing range, in panel A. No significant interactions were found in the counting range of the dot enumeration task nor in the digits comparison task.

360) = 7.2, p < .001, Partial eta-squared = 0.04 (numerosity 1: M = 923, SE = 30.6; numerosity 2: M = 1202, SE = 41.6; numerosity 3: M = 1677, SE = 42.4). Also, a significant interaction between group and numerosity was found, F(2, 360) = 72.4, p < .001, Partial eta-squared = 0.29. The Atypical Subitizing group showed significantly longer RTs in numerosities 2, F = 10.2, p = .038; and 3, F = 100, p < .001. See Fig. 1, leftmost panel. A significant effect of age was found in this analysis: F(1, 180) = 5.5, p = .04, Partial eta-squared = 0.03. No significant interaction between group, age and numerosity was found.

An ANCOVA conducted in the counting range's reaction times yield significant group effect: F(1, 178) = 25, p < .001, Partial eta-squared = 0.12 showing the atypical subitizers group significantly higher reaction times (Typical subitizers: M = 3341, SE = 121; Atypical subitizers: M = 4458, SE = 180). Also a numerosity effect was found: F(2, 356) = 11.6, p = .001, Partial eta-squared = 0.06 (numerosity 6: M = 3370, SE = 98.2; numerosity 7: M = 3793, SE = 108; numerosity 8: M = 4535, SE = 168). No significant interaction between group and numerosity was confirmed (see Fig. 1, middle panel). A significant effect of age was found in this analysis: F(1, 178) = 35.3, p < .001, Partial eta-squared = 0.17. Also, a significant interaction between age and numerosity was found: F(2, 356) = 5.3, p = .03, Partial eta-squared = 0.03.

Finally, An ANCOVA conducted in the digits comparison task, including group as between factor, numerical distance (American Psychiatric Association, 2014; Andersson and Östergren, 2012; Ansari et al., 2007) as within factor and age as continuous predictor, showed significant group effect: F(1, 180) = 5.4, p = .005, Partial eta-squared = 0.03, exhibiting the atypical subitizers significantly longer reaction times (Typical subitizers: M = 1866, SE = 149; Atypical subitizers: M = 2501, SE = 223). No significant distance effect nor interaction between group and numerosity were found. However, detailed planned comparisons showed there were significant differences between reactions times in distance 1 compared to distance 2 and to distance 3; p = .04, but not between distance 2 compared to distance 3 (distance 1: M = 2360, SE = 145; distance 2: M = 2134, SE = 138; distance 3: M = 2058, SE = 164). Likewise, the atypical subitizer's group exhibited significantly higher reaction times for numerical distance 1: p = .03 and distance 2: p = .04, compared to typical subitizers, but both groups showed similar reaction times for distance 3, which may explain the lack of interaction between group and numerical distance in this analysis. Like in the previous analysis, a significant effect of age: F(1, 180) = 42.4, p < .001, Partial eta-squared = 0.19 and a significant interaction between age and numerical distance was found: F(2, 360) = 3.6, p < .03, Partial eta-squared = 0.02.

5.3. Behavioral profiles analysis

Due to some of the groups resulting from combining the BNB and EM classification criteria were too small to perform group analysis, it was decided to include in the ANCOVAS comparing the reaction times in the subitizing and counting ranges of the dot enumeration task and the digits comparison task between the Controls_T and Controls_A groups and the LAA_T and LAA_A vs. Controls_T group. See groups' sample size resulting from combining the BNB and EM classification criteria in Table 3.

5.4. Controls_T vs. Control_A groups

ANCOVAs conducted in the subitizing range including group (Controls_T and Controls_A groups) as between factor, age as

Table 3
Sample sizes resulting from combining the BNB and EM classification criteria.

Classification	Count	Percent
Controls_T	100	50.0
ALD_T	12	6.0
DD_CN_T	5	2.5
DD_CP_T	4	2.0
Controls_A	25	12.5
ALD_A	15	7.5
DD_CN_A	3	1.5
DD_CP_A	10	5.0
DD_CP_CN_A	1	0.5
ALD_DD_CN_T	2	1.0
ALD_DD_CP_T	2	1.0
ALD_DD_CN_A	3	1.5
ALD_DD_CP_CN_A	1	0.5
Subjects excluded due to noisy RT patterns	17	8.5
Total	200	100

Note: Controls_T: Controls with typical subitizing performance, Controls_A: Controls with atypical subitizing performance. LAA_T: Low Arithmetic Achievement Group with typical subitizing performance, LAA_A: Low Arithmetic Achievement Group with atypical subitizing performance, DD_T: Developmental Dyscalculia Group with typical subitizing performance, DD_A: Developmental Dyscalculia Group with atypical subitizing performance.

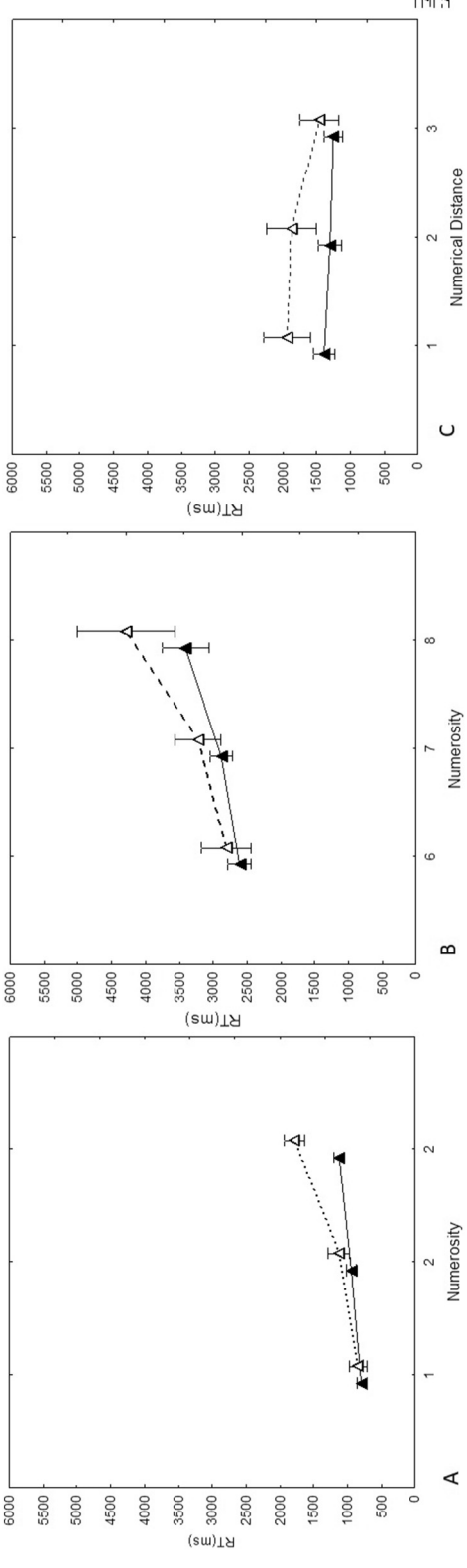


Fig. 2. Subitizing, counting (dot enumeration) and numerical magnitude comparison (digit comparison) processes in the typical controls: (Controls_T, $N = 100$) and atypical controls regarding subitizing ability (Controls_A, $N = 25$) and atypical subitizing group (non-subitizers). Significant numerosity effects were found in the subitizing and counting ranges. However, no significant distance effect was confirmed. Note the significant numerosity per group interaction in the subitizing range (panel A). No significant interactions were found in the counting range of the dot enumeration task nor in the digits comparison task.

continuous predictor and numerosity as within factor, showed significant effect of group: F(1, 122) = 16.2, p < .001, Partial eta-squared = 0.12. The Controls_A group exhibited significantly higher reaction times compared with Controls_T (Controls_T group: $M = 954$, $SE = 31.3$, Controls_A group: $M = 1252$, $SE = 65.4$). Significant numerosity effect: F(2, 244 = 9.1, p < .001, Partial eta-squared = 0.07, was found (numerosity 1: $M = 821$, $SE = 36$; numerosity 2: $M = 1035$, $SE = 43$; numerosity 3: $M = 1452.26$, $SE = 41.88$), and also, an interaction between numerosities and group: F(2, 244) = 42.3, p < .001, Partial eta-squared = 0.26. According to planned comparisons analysis performed, there were statistically significant differences between the groups in numerosities 2, $F = 4.4$, $p = .04$ and 3, $F = 56.6$, p < .001; showing the Controls_A group significantly higher RT. A significant effect age was also found: F(1, 122) = 5.2, p = .04, Partial eta-squared = 0.04). See Fig. 2, panel A.

Concerning the counting range, the ANCOVA including group (Controls_T and Controls_A groups) as between factor, age as continuous predictor and numerosity as within factor (Fig. 2, panel B), showed significant effect of group: F(1, 122) = 4.4, $p = .04$, Partial eta-squared = 0.04. The Controls_A group exhibited significantly higher reaction times compared with Controls_T (Controls_T group: $M = 2968$, $SE = 94.4$, Controls_A group: $M = 3439$, $SE = 197$). Significant numerosity effect: F(2, 244) = 11.4, p < .001, Partial eta-squared = 0.09, was found (numerosity 6: $M = 2708$, $SE = 102$; numerosity 7: $M = 3053$, $SE = 93.2$; numerosity 8: $M = 3849$, $SE = 197$). No interaction between numerosities and group: was found. A significant effect age was also found: F(1, 122) = 39.7, p < .001, Partial eta-squared = 0.25.

In the digits comparison task (Fig. 2, panel C), a significant group effect was found: F(1, 122) = 8.1, p = .005, Partial eta-squared = 0.06; showing the Controls_A group, significantly higher RT compared to Controls_T (Controls_T group: $M = 1313$, $SE = 65.4$, Controls_A group: $M = 1755$, $SE = 136$). No numerical distance effect nor interaction between numerical distance and group were found. A significant effect age was also found: F(1, 122) = 12.6, $p < .001$, Partial eta-squared = 0.09.

5.5. Controls_T vs. the LAA_T and LAA_A groups

The ANCOVA contrasting the reaction times among the Controls_T and the LAA_T and LAA_A groups (between groups effect) in the subitizing range (Fig. 3, panel A), showed significant group effect F(2, 123) = 17.2, p < .001, Partial eta-squared = 0.22, showing Controls_T group significantly lower reaction time compared to LAA_T and LAA_A groups, but no significant RT difference was detected between this two groups. Also, numerosity F(2, 246) = 10.2, p < .001, Partial eta-squared = 0.08 and interaction between group and numerosity, F(4, 246) = 28.3, p = .001, Partial eta-squared = 0.32 effects were found. Planned comparisons showed Controls_T exhibited significantly lower RTs compared to LAA_T group in numerosity 1, $F = 22.2$, p < .001, numerosity 2, $F = 11.1$, p < .001 and 3, $F = 14.3$, p < .001. A significant effect of age was also found, F(1, 123) = 5.3, p = .04, Partial eta-squared = 0.04.

Additionally, Controls_T showed no significant difference in RTs compared to LAA_A group in numerosity 1, p = .37, but showed significantly lower reaction times in numerosities 2, $F = 4.6$, p = .03 and 3, $F = 75.2$, p < .001. Also, significant differences in RT were found between LAA_T and LAA_A groups in numerosities 1, F = 9.4, p = .003 and 3, F = 14.7, p < .001, but no in numerosity 2, p = .33.

In the counting range (Fig. 3, panel B), the ANCOVA contrasting the reaction times among the Controls_T and the LAA_T and LAA_A groups (between groups effect) showed significant group effect F(2, 121) = 22.4, p < .001, Partial eta-squared = 0.27. The Controls_T group exhibited significantly lower RT compared both, to LAA_A and LAA_A groups. Also, the expected numerosity effect was observed F(2, 242) = 9.2, p < .001, Partial eta-squared = 0.07. No interactions

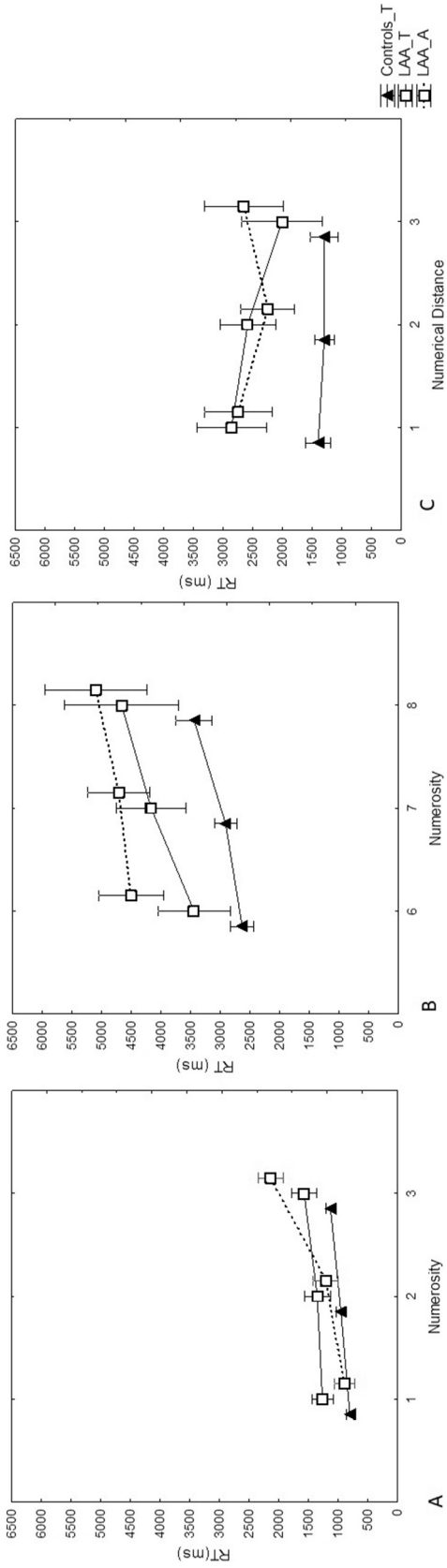


Fig. 3. Subitizing, counting (dot enumeration) and numerical magnitude comparison (digit comparison) processes in the typical controls: (Controls_T, $N = 100$) and Low Arithmetical Achievement typical (LAA_T, $N = 12$) and atypical subitizing group (non-subitizers, LAA_A, $N = 15$). Significant numerosity effects were found in the subitizing and counting ranges. However, no significant distance effect was confirmed. Note the significant numerosity per group interaction in the subitizing range (panel A). No significant interactions were found in the counting range of the dot enumeration task nor in the digits comparison task.

between group and numerosity were found in the counting range. The age effect was found as expected also in this range of the exact enumeration task, $F(2, 121) = 36.9$, $p < .0.001$, Partial eta-squared = 0.23.

In the digits comparison task (Fig. 3, panel C), the ANCOVA contrasting the reaction times among the Controls_T and the LAA_T and LAA_A groups (between groups effect) showed significant group effect: $F(2, 123) = 18.0$, $p < .001$, Partial eta-squared = 0.23. The Controls_T group exhibited significantly lower RT compared both to LAA_A and LAA_A groups. No distance effect nor interactions between the numerical distance and group were found in the analysis. The lack of distance effect suggests a ceiling effect in performance in the task. Finally, an age effect was also found in the task $F(1, 123) = 7.6$, $p = .0.007$, Partial eta-squared = 0.06.

6. Discussion

In the present study, it was hypothesized that arithmetic achievement is the result of the contribution of relatively independent neurocognitive networks underlying exact numerosities representation, numerical symbolic and non-symbolic magnitudes comparison and of domain general cognitive processes, including visual attention mechanisms for the representation of small numerosities. In line with this prediction, several patterns of associations of deficits were detected when applying the BNB standardized test to diagnose Developmental Dyscalculia, defined by a significant low performance in basic numerical abilities tasks and differentiate it from low arithmetical achievement, understood as a significant difficulty in performing and/or retrieving arithmetic facts.

These findings support that relatively independent neurocognitive mechanisms (visual attention involved in small numerosities exact estimation vs. counting) may produce distinct profiles at the behavioral level, and support the idea proposed by previous studies focused in the description of heterogeneity in Developmental Dyscalculia following a similar group design as the one presented here (Skagerlund and Träff, 2016) or following a more suitable case study methodology (Träff et al., 2016), that the origins of different profiles of mathematical deficits in children with DD diverge. Unfortunately, no other domain-general cognitive nor domain-specific cognitive measures were available for the sample included in the present study and it was not possible to conduct specific analysis regarding the interactions between the numerical processes assessed here and other specific or general cognitive process.

Different patterns of associations and dissociations of deficits among subitizing, counting and numerical magnitude comparison abilities have been reported in subjects with acquired dyscalculia, genetic syndromes exhibiting developmental dyscalculia as part of their cognitive profile and children with DD. Spared subitizing and impaired counting have been found in brain damaged demented subjects (Demeyere, Lestou, and Humphreys, 2010; Seron et al., 1991) in patients with simultagnosia (Dehaene and Cohen, 1994) and in a patient with severe acquired dyscalculia (Cipolotti, Butterworth, and Denes, 1991). Spared subitizing and impaired counting processes were reported in a patient with aphasia and semantic dementia together with similarly impaired counting processes and impaired subitizing in a patient with Gerstman Syndrome (Lemer, Dehaene, Spelke, and Cohen, 2003). Impaired subitizing and spared counting and magnitude comparison were described in patients with Turner Syndrome by Bruandet et al. (2004). Patterns of performance showing the opposite dissociation were described in children with velocardiofacial syndrome: spared subitizing and impaired counting (Simon et al., 2005) and spared subitizing together with impaired magnitude comparison (Simon et al., 2005; De Smedt et al., 2007). Several studies carried out on children with arithmetic learning disabilities have reported poor performance on subitizing (Koontz and Berch, 1996), counting (Geary, Bow-Thomas, and Yao, 1992), or magnitude comparison tasks (Geary et. al, 2000; Rousselle and Noël, 2007). Likewise, others have reported association of deficits

among these processes: deficits in subitizing and magnitude comparison processes (Reeve and Reynolds, 2004) and the combination of subitizing, counting and magnitude comparison deficits (Landerl, Bevan, and Butterworth, 2004; Reigosa-Crespo et al., 2012). Additional evidence supporting the heterogeneity of the neurocognitive foundations of mathematical achievement has been reported by several neuroimaging studies (Piazza, Mechelli, Butterworth, and Price, 2002; Piazza, Giacomini, Le Bihan, and Dehaene, 2003; Cutini, Scatturin, Moro, and Zorzi, 2014; Demeyere, Rotshtein, and Humphreys, 2012; Demeyere, Rotshtein, and Humphreys, 2014; Cantlon, Brannon, Carter, and Pelphrey, 2006; Mussolin, Mejias, and Nöel, 2010; Ansari et al., 2007; Vetter, Butterworth, and Bahrami, 2010).

On the other hand, a group of typically achieving children according to the mental calculation item-timed task, but exhibiting aberrant subitizing patterns, were found here. To the best of our knowledge, this is the first study reporting significantly slow subitizing in a group of typically developing controls in terms of arithmetic fluency. Specifically, Controls_A exhibited significantly slower RTs for numerosities two and three, compared to typically developing controls; which suggests this group showed a very reduced subitizing range (to one object), or even that this children directly rely in the verbal counting process when exact enumeration of sets is required. Also, it may indicate that probably exact enumeration is carried out at the expense of other compensatory mechanisms, likely involving executive functions and/or a combination of strategies (e.g.: pattern recognition and addition operations). In contrast, no significant interactions were found between numerosity and group in the counting range, or in the digits comparison task; which suggest similar cognitive mechanisms are underlying performance in the corresponding tasks.

The fact that impaired subitizing can coexist with spared arithmetic fluency challenges the defective number module hypothesis (Butterworth, 2005) which proposes that a highly specific deficit of an innate capacity to understand and represent quantities leads to difficulties in learning number and arithmetic. However, note the classification employed here depends upon the performance of a mental calculation task, which, although based in population norms, may not sufficiently sample the kind of arithmetical complexity that children encounter in everyday activities, at school or otherwise, nor represent the type of task that may directly benefit from the contribution of typical subitizing processes. There is no consensus regarding if subitizing may be influenced by domain-general abilities, such as executive functions or whether this domain general processes might mediate or moderate the contribution of subitizing to arithmetic (Kroesbergen et al., 2009). On the other hand, other domain general abilities have been reported to contribute differently to numerical processing; for instance, subitizing requires more attentional resources than does numerosity estimationin in adults (Burr et al., 2010), and also general domain processes may impact in other processes (working memory is important to serial counting but not to subitizing, Ashkenazi (2016)) that also interact and support arithmentic achievement. Unfortunately, as previously the unavailability of complementary domain-general and domain-specific cognitive measures prevents to draw definitive conclusions from the present results. More research is required in order to elucidate the specific mechanisms underlying arithmetical achievement and the interactions between the various contributing cognitive processes involved.

On the other hand, according to the comparison among LAA_T, LAA_A vs. typically subitizing controls, the results presented here suggest children with low arithmetic achievement exhibiting typical subitizing abilities show significantly slower but qualitatively similar subitizing abilities compared to control children, whereas children with low arithmetic achievement exhibiting atypical subitizing showed significantly lower reaction time in numerosity 1 (similar to the Controls_T group) compared to the LAA_T group but significantly higher RT for numerosity 3 compared to both Controls_T and LAA_A groups, which suggests specific difficulties when processing numerosity 3 in this group, but not a general slowing of processing speed when it comes to process exact small numerosities, as seems to be the problem for the LAA_T group. Again, no significant interactions were found in the counting range of the dots enumeration task, nor in the digits comparison task, although in both cases ALD children performed significantly slower, which suggest these children may be experiencing a slowing on information processing hindering arithmetic fluency and mathematical achievement; which in turn might indicate more immature or less automatized access to numerical representation and/or deficits in the interphase linking analogic and numeric symbolic representations and its underlying neural networks, given both tasks included stimuli and/or answers to be provided in Arabic symbolic format. Note that all the analysis were conducted on the reaction times corrected by the processing speed of the subjects, as assessed by the simple reaction time task of the BNB Battery, which suggest it may rather be a slowing in the processing of numerical material than a more general processing speed deficit.

7. Conclusions

The results of the present study support the heterogeneity in the neurocognitive foundations and cognitive profile of Developmental Dyscalculia and suggest the origin of the disorder may result from the contribution of domain-specific numerical processes and its interactions with other domain general processes. Additionally, the results suggest children with varying degrees of numerical and arithmetic performance exhibiting atypical subitizing abilities are not only significantly slower, but rely on compensatory mechanisms and strategies, compared to typical subitizers.

In our view, the study of the neural foundations of basic numerical processing should be grounded in the identification of subtypes with characteristic cognitive profiles. Accordingly, in the present study the sample classification was conducted following a theoretically oriented approach that considers dyscalculia as the expression of a deficit in the development of basic numerical capacities (Butterworth, 2003). This approach emphasizes the characterization of the processes involved in the cognitive deficit (contrary to the more traditional diagnosis, based on arithmetic performance) and therefore allows the study of different subtypes within the disorder. This procedure for individual subjects classification could allow the design of more specific and effective intervention strategies. Nevertheless, as was previously mentioned, it was not possible to conduct a detailed neurocognitive assessment of the sample, and therefore it was not possible to analyze further cognitive underpinnings of the behavioral profiles described here. Future studies should include a fine-grained assessment of domain-general and domain-specific cognitive processes in order to contribute to the models of numeric processing and calculation.

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